Motivational introduction to agent-based models

The co-evolutionary model
of the financial market
on the dynamical network

Simulations replacing microscopes

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Outline

1. **The general concept of agent-based and multi-agent models, distributed inteligence**
2. **Agent based approach and econophysics** www.unifr.ch/econophysics/
3. **Example: Application to the stock market**
   - **Further motivations - leave lattice, towards flexible topology**
   - **Introduction to the spin stock market models**
   - **Co-evolutionary network version of the model** (complex network topology, spin decoration on the complex net, construction of agent's intranet (econo-brain), spins replaced by the spin modules, model build-up, formalism of the co-evolutionary dynamics, single agent operators, local field concept, magnetization related to the log-price returns, local fitness and minority game concept, slow adaptivity to the prototype subleader, slow dynamics of the network reconnections, extremal selective dynamics)
   - **Numerical analysis** meta-optimized parameters, statistics - scaling of price returns, identification of the basic time scale, network of the distibuted (sub)leaders, statistics of the fitness and node degrees, clustering coefficient
4. **Conclusions**

The general concept of agent-based and multi-agent models, distributed inteligence

Applications: robotics, nanoscience, social and evolutionary computing, biology, medicine (cancer models), economy (price formation, organization structures and virtual trading)

Definition (*IBM*): Intelligent agents are **software entities** that carry out some set of operations on behalf of a user or another program with some **degree of independence** or autonomy, and in so doing, employ some knowledge or representation of the user's **goals or desires**.

Properties of inteligent agent (Wooldridge, 1995, 2002)

Autonomy - without direct (instant) access or plan of programmer

Social ability - interactions with neighbours

Reactivity to environment

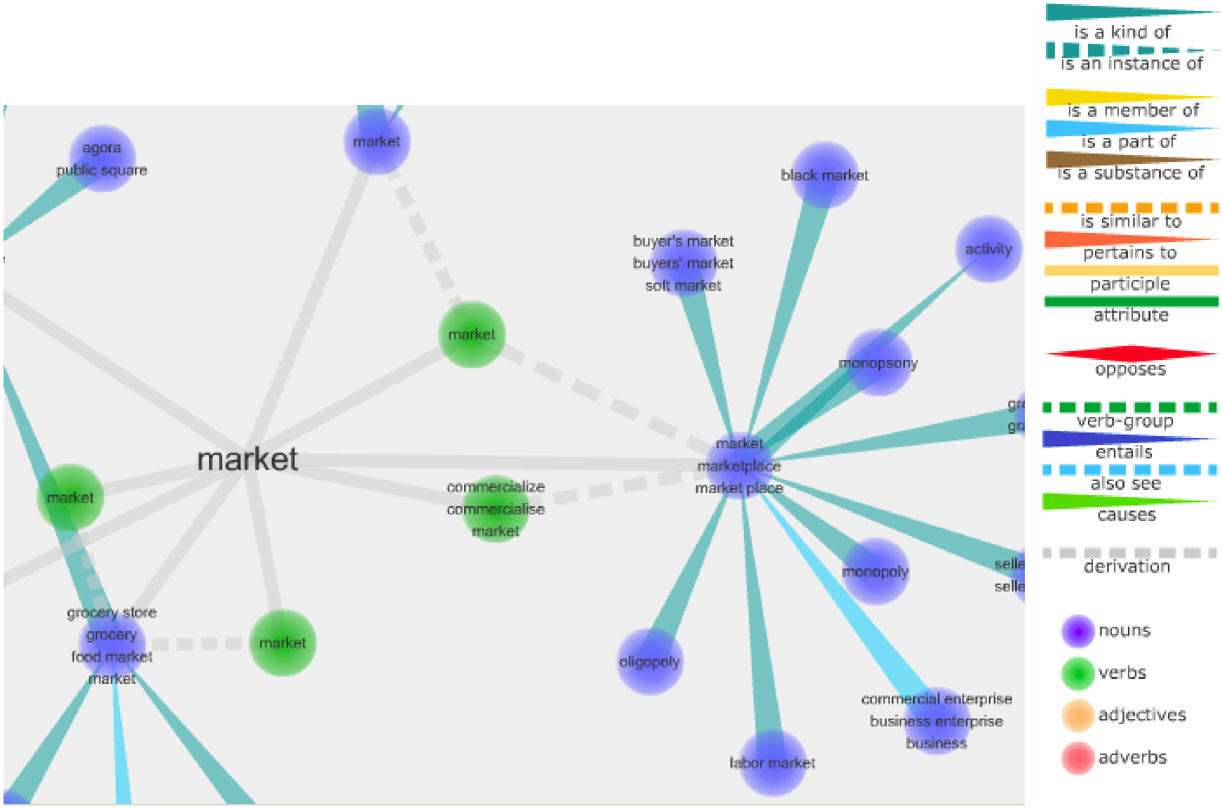
Pro-activeness (Proactive behavior involves acting in advance of a future situation, rather than just reacting). The property is called as well Anticipation (ACS)).

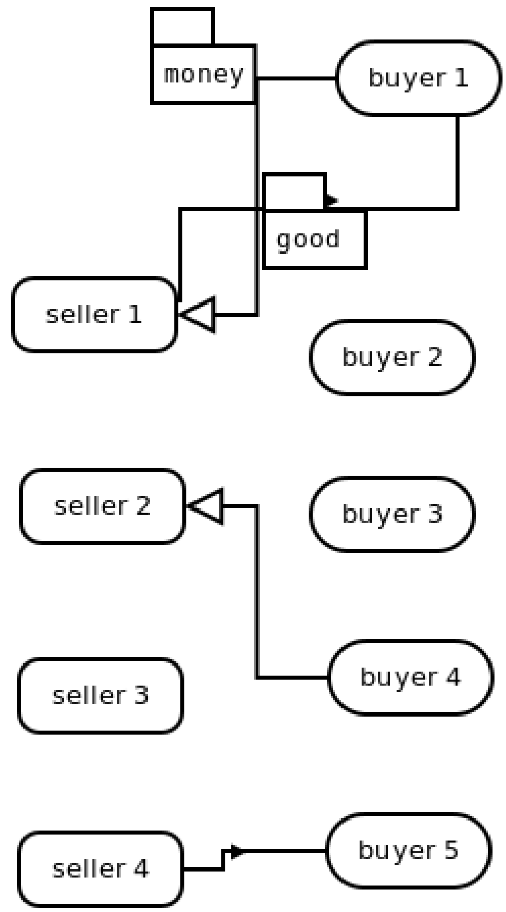
Where is the difference between agent and multi-agent system (MAS)?

Example: design of market architecture by means of multi-agent system

Definition: *A market is one of many varieties of systems, institutions, procedures, social relations and infrastructures whereby parties engage in exchange. While parties may exchange goods and services by barter, most markets rely on sellers offering their goods or services (including labor) in exchange for money from buyers. It can be said that a market is the process in which the prices of goods and services are established.*

http://www.visuwords.com/




money
buyer 1
good
seller 1
buyer 2
seller 2
buyer 3
seller 3
buyer 4
seller 4
buyer 5

```
Environment  :          market institution
Population of agents: { A(1),A(2), ...  B(1),B(2), ... C(1),C(2), ...D(1),D(2), ...}
.................................................................................
Agents: Agent type A  : seller   :   attribute:   offer(s), rivalry effort,
                                                  distribution channel(s),
                                                  owned money, owned good
        Agent type B  : buyer    :   attribute:   order(s),owned money, owned good
        Agent type C  : employer :   attribute:   work offer, owned money
        Agent type D  : employee :   attribute:   duty, owned money

Links- connections among mediators (may be percieved as agents as well):
..............................................................................
      link type a : money flow    ;  attribute:  currency
      link type b : good  flow    ;  attribute:  intensity = amount per time unit
                                                 price and price elasticity, quality
      link type c : debt  flow    ;  attribute:  intensity, currency
      link type d : labor flow    ;  attribute:  intensity, quality
      link type e : service flow  ;  attribute:  quality

Examples of interaction processes (so called algo-chemistry of agents):
.......................................................................
        i,j names of agents
       ...............................................................................
        A(i), B(j)  interaction event (contract) mediated by a(money flow), b(good flow)
       [A(i), B(j); under (a,b)]  |--- reaction ----->  A'(i), B'(j)
        C(i), D(j)  interaction event (contract) mediated by a(money flow), d(labour flow)
       [C(i), D(j); under(a,b)]   |--- reaction ----->  C'(k), D'(l)
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Example: **Agent-based model of the collaborative knowledge building.**

The knowledge of ith agent is represented stored by tuple including certain types of properly quantified (normalized) knowledge (e.g. different categories, skills) $(W_{i1}, W_{i2}, \ldots, W_{in})$.

Update of i-th agent knowledge is realized by the opening of three probabilistic (by Monte-Carlo) channels:

(1) With probability p_{pair} we open pair-wise social contact. The learning process is described by the rule

$$W_{ik} \leftarrow W_{ik}(1 - \eta_{(ij),k}) + W_{jk}\eta_{(ij),k} \,. \tag{1}$$

Here the social network is defined by means of plasticity $\eta_{(ij),k}$, which mediates the knowledge (kth category) transfer from the knowledge provider j to the knowledge receiver i.

(2) With probability p_{comp} the ith agent learns the overall (global) instant knowledge hold by the company (e.g. local wiki pages)

$$W_{ik} \leftarrow W_{ik}(1 - \eta_k^{\text{G}}) + \eta_k^{\text{G}} \left(\frac{1}{n} \sum_{s=1, s\neq k}^{n} R_s R_k W_{sk} \right) , \tag{2}$$

where $\eta_k^{\text{G}} \in (0,1)$ learning plasticity of kth category attained within the company, where R_k, R_s are the local ratings of the category (information), or relevance, veracity of the source - experience of the worker, respectively.

(3) With probability $1 - p_{\text{pair}} - p_{\text{comp}} > 0$ the agent learns from the environment by the rule

$$W_{ik} \leftarrow W_{ik}(1 - \eta_k^{\text{E}}) + \eta_k^{\text{E}} W_k^{\text{E}} \,, \tag{3}$$

where $\eta_k^{\text{E}} \in (0,1)$ is the learning plasticity of the kth category attained by the interaction with the environment represented by Internet.

Agent based approach and econophysics

www.unifr.ch/econophysics/

- Analysis of the standard models of statistical physics that are minimalist in the parametrizations but reach in the properties and description of reality:

  (a) bottom-up construction and principles;

  (b) specification of small-scale entities-degrees of freedom (atoms, spins, molecules, ...);

  (c) definition of interaction(s) between entities from (b);

  (d) consequences, expectations, predictions (i.e. calculation) of large-scale, long-time statistics [focus on emergence and unexpected phases; of special relevance are some critical points];

- (economic) reinterpretation of the entities (spins are replaced by the market agents, flip dynamics is interpreted as dynamics of sell-buy orders)
- simulation: (parallelelized - computer cluster Gentoo linux, openMosix [single-system image clustering]; MPI tools;]
- analysis of the large-scale and long-term consequences - averages, mean values related to the measurements, observations

Emergence: appearence of a new quality or feature - basin of attraction, attractor

Emergence: the way complex systems and patterns arise out of a multiplicity of the relatively simple interactions.

Integrative level: or level of organization, is a set of phenomena emerging on pre-existing phenomena of lower level.

Attractor: is a set towards which a variable moving according to the dictates of a dynamical system evolves over time.

Spontaneous symmetry breaking is a spontaneous process by which a system in a symmetrical state ends up in an asymmetrical state.

SZNAJD MODEL AND ITS APPLICATION TO POLITICS

Mehmet Oz

May 5, 2008

ABSTRACT: Sznajd model of opinion formation in socities has found many applications, primarily in politics. Here, we review the basic model, show that its natural extension to two dimensions exhibits a phase transition, and then focus on its application to proportional elections where our model predicts the exponent for the power law distribution of number of votes obtained by different candidates in Brazilian proportional elections.

In search for a simple model of opinion formation, Katarzyna Sznajd-Weron and Josef Sznajd considered a society where each individual time and again has to decide between two choices (A or B)[3]. We note that this setting applies particularly well for referendums, where electorate vote 'Yes' or 'No'. To keep the model simple, it was also assumed that the society in question was a closed society, meaning that from the first timestep the votes are taken until the last timestep, there is no inclusion or exclusion to the electorate; the same people vote over and over again. The concept of a closed society can be interpreted as analogous to an isolated system in physical language. The final assumption that the model had to fit was that the steady states of the model are the three limiting cases:

(a) Each person votes for A

(b) Each person votes for B

(c) Equal number of people vote for A and B

The model consists of individuals, identified as magnets or spins, being ordered on a one dimensional Ising chain. Hence, each individual is regarded as a spin variable S_i where $S_i = 1$ corresponds to voting for A, and $S_i = -1$ corresponds to voting for B. The spins evolve in time according to the following dynamic protocol[4]:

(a) In each timestep, a pair of neighboring spins S_i and S_{i+1}, henceforth which we will refer as a bond, is chosen randomly to interact with their two nearest neighbors S_{i-1} and S_{i+2}.

(b) If $S_i = S_{i+1}$ then $S_{i-1} = S_i$ and $S_{i+2} = S_i$.

(c) If $S_i = -S_{i+1}$ then $S_{i-1} = S_{i+1}$ and $S_{i+2} = S_i$.

rules (simple reactive agents)

$$\text{if } (S_i = S_{i+1}) \text{ then } S_{i-1} \leftarrow S_i, \quad S_{i+2} \leftarrow S_i$$

$$\text{if } (S_i = -S_{i+1}) \text{ then } S_{i-1} \leftarrow S_{i+1}, \quad S_{i+2} \leftarrow S_i$$

```c
i = (int) (drand48 () * L);

ip1 = i + 1;
ip2 = i + 2;
if (ip1 == L)
  ip1 = 0;

if (ip2 == L)
  ip2 = 1;

im1 = i - 1;
im2 = i - 2;
if (im1 == -1)
  im1 = L - 1;

if (im2 == -1)
  im2 = L - 2;

if (S[i] == S[ip1])
  {
    S[im1] = S[i];
    S[ip2] = S[i];
  }

if (S[i] == -S[ip1])
  {
    S[im1] = S[ip1];
    S[ip2] = S[i];
  }
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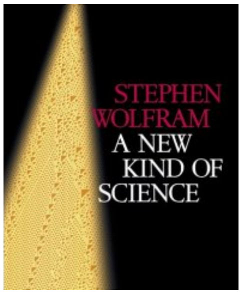
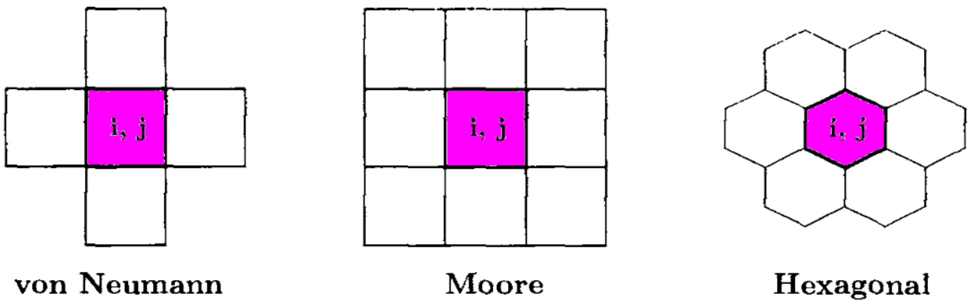
Further motivations -from lattice towards flexible topology

CA rules



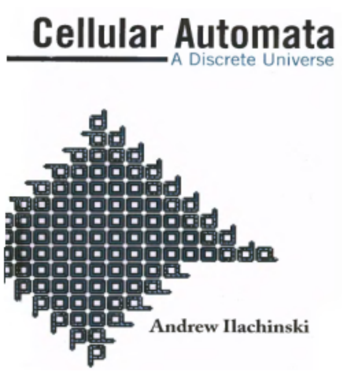


2D dynamical rule (von-Neumann neighbourhood)

$$\sigma_{i,j}^{(t+1)} = \varphi\left(\sigma_{i,j}^{(t)}, \sigma_{i-1,j}^{(t)}, \sigma_{i+1,j}^{(t)}, \sigma_{i,j-1}^{(t)}, \sigma_{i,j+1}^{(t)}\right), \qquad \sigma_{i,j}^{(t)} \in \{0, 1, 2, \ldots, K\}$$

$$\Pi^{(t)} = \Big| \ \{\sigma_{i,j}^{(t)}\} \ \Big\rangle$$

Structuraly dynamic CA (SDCA) rules



$$\sigma_i^{(t+1)} = \Theta\left(\alpha_i + \sum_{k=1}^{L} \beta_{i,k}\, \ell_{i,k}^{(t)}\, \sigma_k^{(t)}\right), \qquad \sigma_i^{(t)} \in \{0,1\} \tag{4}$$

$$\ell_{i,j}^{(t+1)} = \Theta\left(a_{i,j} + \sum_{k=1}^{L} b_{i,j,k}\, \ell_{j,k}^{(t)} \sigma_k^{(t)}\right), \qquad \ell_{i,j}^{(t)} \in \{0,1\} \tag{5}$$

$$\Pi^{(t)} = |\ \{\sigma_{i,j}^{(t)}\}\ ,\ \{\ell_{i,j}^{(t)}\}\ \rangle$$

Introduction to spin stock market models

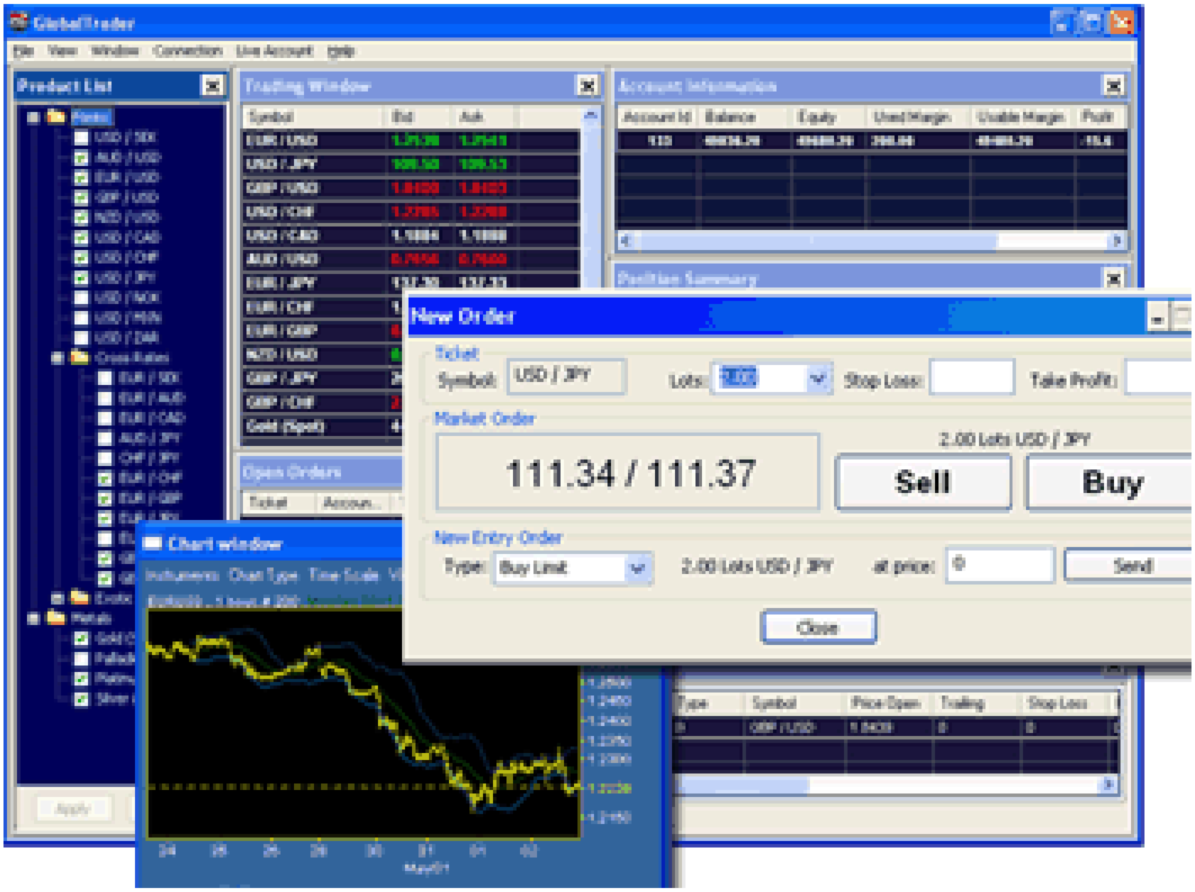
Spin-Lattice approaches

- R.Cont, J.P.Bouchaud, in J.Bouchaud and M.Potters, Theories des Risques Financiers (Alea Saclay/Eyrolles, 1997)
- D.Chowdhury, D.Stauffer, European Physical Journal B, **8** (1999) 477
- S.Bornholdt, Int. J. Mod. Phys.C **12** (2001) 667
- T.Kaizoji, S.Bornholdt, Y.Fujiwara, Physica A **316** (2002) 441
- T.Takaishi, cond-mat/0503156

....Network approach:

- D. Horváth, Z. Kuscsik, M. Gmitra, "The co-evolutionary dynamics of directed network of spin market agents", Physica A, **369** (2006) 780.

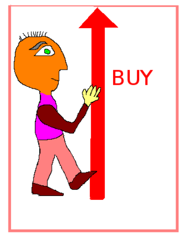
Trading Software



Ising and Blume-Capel spin $S = 1$ family of models

- elementary physical models of N-body problem

- i-th agent **spin encodes**:

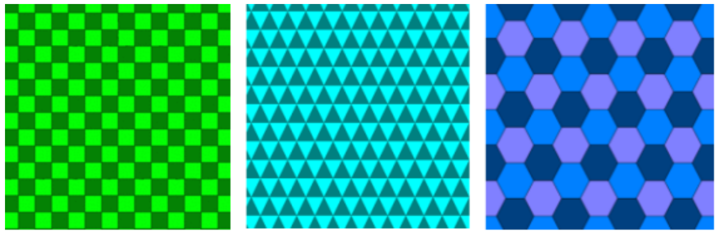


i-th agent **spin encodes**:
BUY order $S^{(t)}(i) = +1$
SELL order $S^{(t)}(i) = -1$
STAY inactive $S^{(t)}(i) = 0$

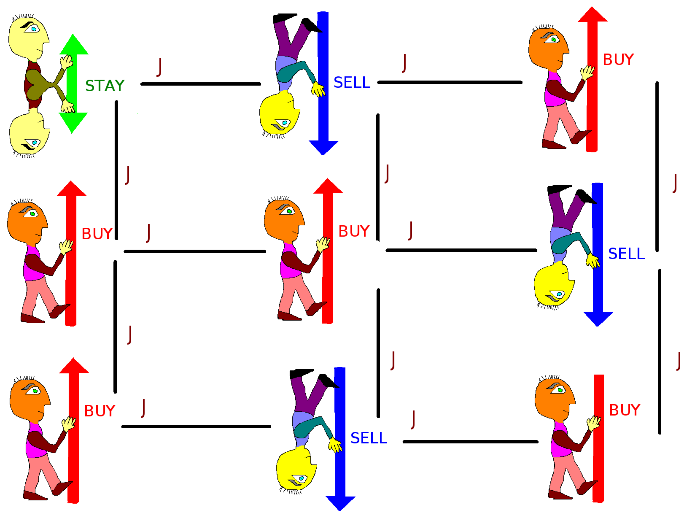
- provide very complex spatio-temporal behavior

- recent efforts - similarity between fluctuations in the economy and N-body problem near the critical point

- **regular lattices**:



- **interactions**: ferro, **spin-glass**, amorphous (AF-F)



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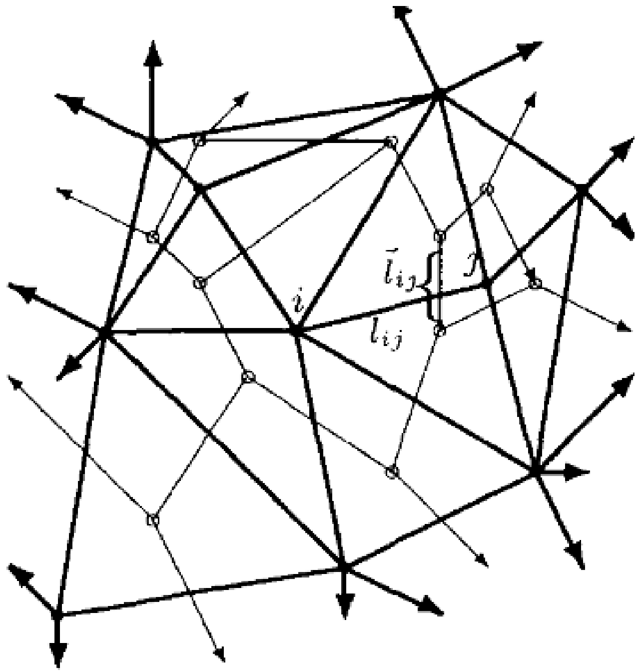


Quantum gravity

PHYSICAL REVIEW E **66**, 016104 (2002)

Ising model on networks with an arbitrary distribution of connections

S. N. Dorogovtsev,[1,2,*] A. V. Goltsev,[2,†] and J. F. F. Mendes[1,‡]
Departamento de Física and Centro de Física do Porto, Faculdade de Ciências, Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal
[2]*A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*
(Received 10 March 2002; published 8 July 2002)

We find the exact critical temperature T_c of the nearest-neighbor ferromagnetic Ising model on an "equilibrium" random graph with an arbitrary degree distribution $P(k)$. We observe an anomalous behavior of the magnetization, magnetic susceptibility and specific heat, when $P(k)$ is fat tailed, or, loosely speaking, when the fourth moment of the distribution diverges in infinite networks. When the second moment becomes divergent, T_c approaches infinity, the phase transition is of infinite order, and size effect is anomalously strong.

DOI: 10.1103/PhysRevE.66.016104 PACS number(s): 05.50.+q, 05.10.−a, 05.40.−a, 87.18.Sn

[1] A.-L. Barabási and R. Albert, Science **286**, 509 (1999
[2] S. H. Strogatz, Nature (London) **401**, 268 (2001).

the co-evolutionary network version of the model

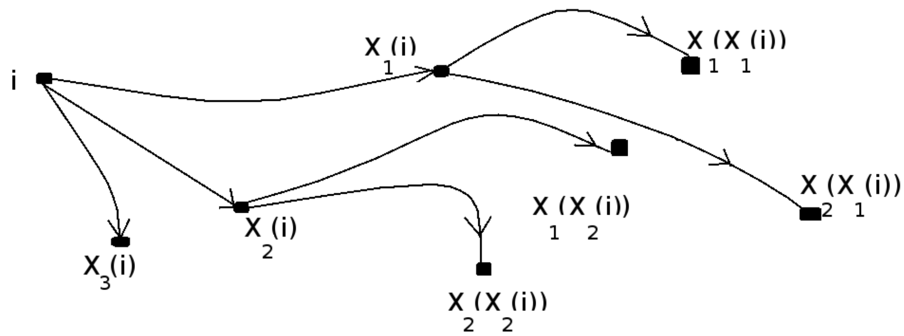
- **main attributes**:

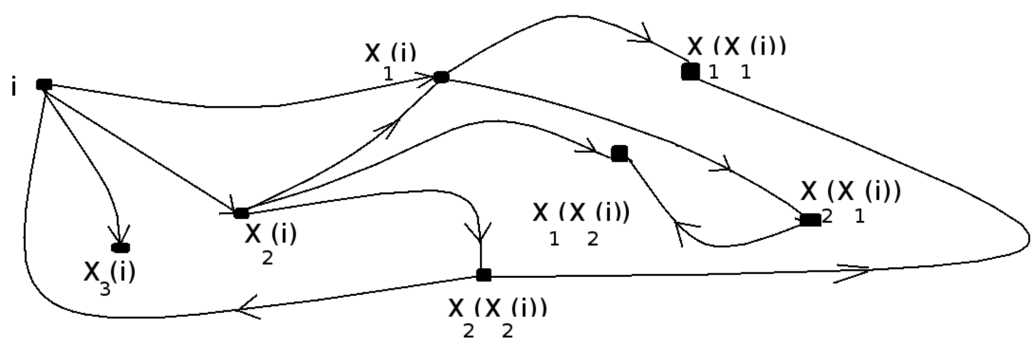
  - **spin state** encodes the agent's **buy-sell order**
  - **minority game** incorporated to **fitness** as an agent's phenotype to valuate [her/his] achievements
  - **extremal selective** dynamics of co-evolving strategies (spices)
  - (**adaptivity**) of strategic variables - couplings

- slow dynamics of network topology **reconnections** using information transfer via **repeated random walk** (RRW)

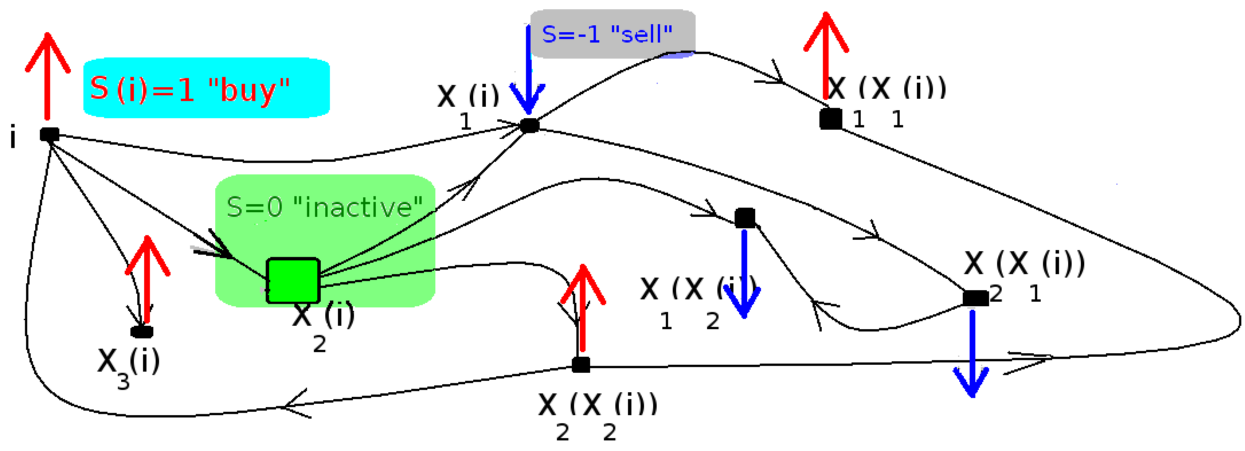
... complex network **topology** ...

- directed **network** of nodes $\Gamma = \{1, 2, \ldots, L\}$.
- node $i \in \Gamma$ is attached via $n = 1, 2, \ldots, N^{\mathrm{out}}$ links to nodes $X_n^{(t)}(i) \in \Gamma$



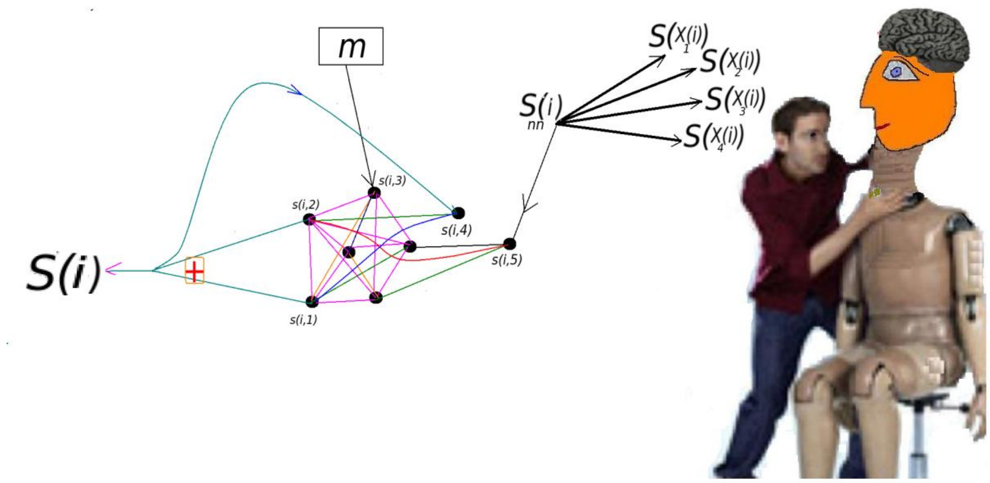


... spin decoration of complex net ...



... construction of agent's intranet ...

- inherent agent's degrees of freedom, model of cognition

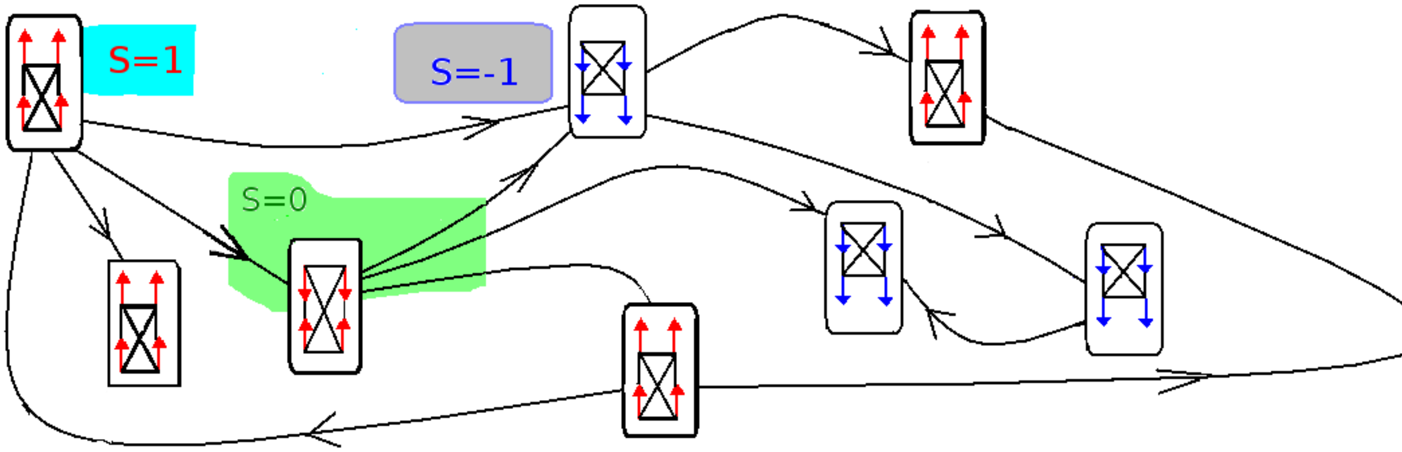


- inputs: all neurons $k \in \{1,2,3,4,5,6,7,8\}$

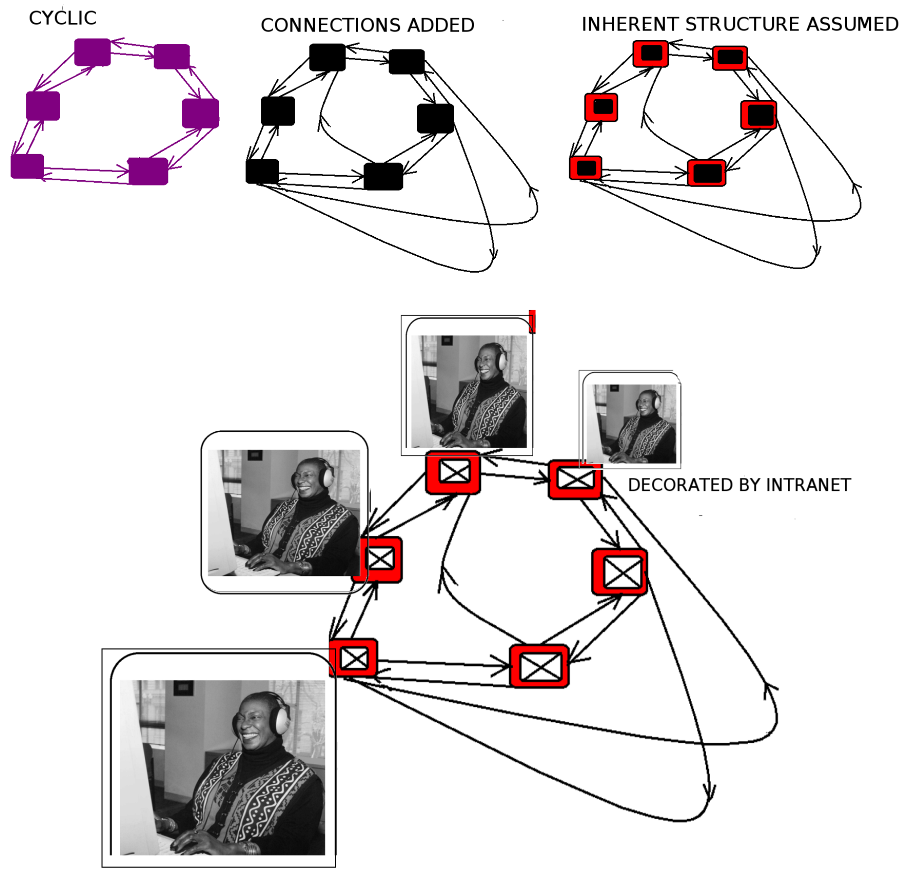
- intranet outputs: selected neurons $k = 1,2$; $s(i,k) \in \{-1,1\}$

macrospin: $S(i) = \frac{1}{2}\ (\ s(i,1) + s(i,2)\ ),\qquad S(i) \in \{-1,0,1\}$

... spins replaced by spin modules ...



...**model build-up**, cyclic subraph guarantees connectivity



... formalism of co-evolutionary dynamics ...

- **stochastic model**

$$\overline{\Pi}^{(t+1)} = \hat{\mathbf{U}}(\overline{\Pi}^{(t)})$$

- **composed configuration**

$$\overline{\Pi}^{(t)} \equiv \left\{ \Pi^{(t)}(1), \Pi^{(t)}(2), \ldots, \Pi^{(t)}(L) \right\} \tag{6}$$

- **single-agent particulars**

$$\Pi^{(t)}(i) \equiv \begin{cases} \text{intranet intra-agent spins} & \Pi^{(t)}_{\text{ss}}(i) \equiv \{ s^{(t)}(i,q) \}_{q=1,\ldots,N_{\text{intr}}} \\ \text{strategic variables} & \Pi^{(t)}_{\text{J}}(i) = \{ \, J^{(t)}_{\text{intr}}(i,k,q) \, \} \\ & \text{where} \;, \;\; q \in \{1, 2, \ldots, N_{\text{intr}} \} \\ \text{network links} & \Pi^{(t)}_{\text{X}}(i) \equiv \{ X^{(t)}_n(i) \} \;, \; n \in I_{\text{out}} \end{cases}$$

… single-agent operators …

	operator(i)	acting on	variables of agent (i)
local field	$\widehat{U}_{\mathbf{ss}}(i)$	acting on	$\Pi^{(t)}_{\mathrm{ss}}(i)$
adaptivity	$\widehat{U}_{\mathbf{Ad}}(i_{\mathrm{a}})$	acting on	$\Pi^{(t)}_{\mathrm{J}}(i_{\mathrm{a}})$
reconnection	$\widehat{U}_{\mathbf{Re}}(i_{\mathrm{r}}, i_{\mathrm{B}})$	acting on	$\Pi^{(t)}_{\mathrm{X}}(i_{\mathrm{r}})$
extremal dynamics	$\widehat{U}_{\mathbf{Ex}}(i_{\mathrm{minF}})$	acting on	$\Pi_{ss}(i_{\mathrm{minF}})\,, \Pi_J(i_{\mathrm{minF}})$ except $X_n(i_{\mathrm{minF}})\,,$ i_{minF}belongs to minimum of fitness $F(i_{\mathrm{minF}}) = \min_{j\in\Gamma} F(j)$

spin flips (changes of sell-buy order) driven by strategic variables

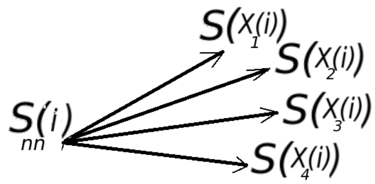
adaptivity of strategic variables based on actual network connections

reconnections driven by fitness and RRW attained information

extremal dynamics

… **local field** $\rightarrow \{$ spin flip $\} \equiv$ $\{$ change of the order sell-buy $\}$ …

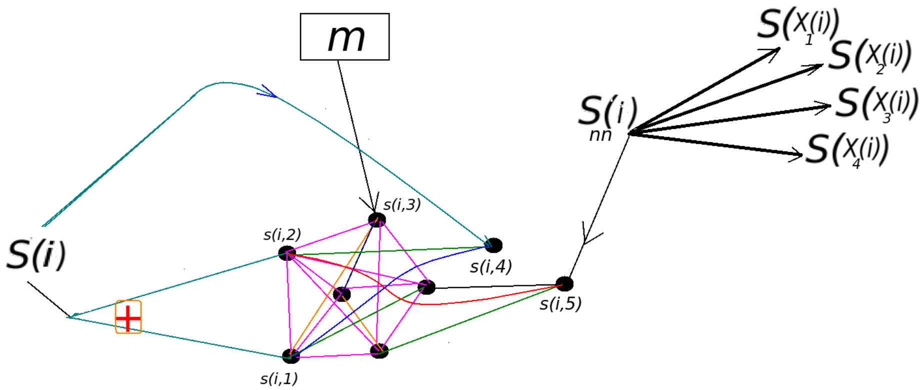
- **local "mean" opinion**



$S^{(t)}_{\rm nn}(i) \leftarrow \frac{1}{N_{\rm out}} \sum_{n \in I_{\rm out}} S^{(t)}(X^{(t)}_n(i))$

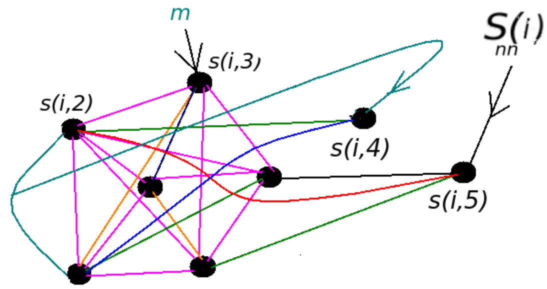
- **inputs of intranet**

$$s(i,3) \leftarrow m\,, \qquad s(i,4) \leftarrow S(i) \qquad s(i,5) \leftarrow S_{\rm nn}(i)$$



- **small-scale intra-agent local field**

$$h_{\rm loc}^{(t)}(i,k) \leftarrow {\rm N}(0,\sigma_{\rm stoch}) + \frac{1}{N_{\rm intr}-1}\sum_{q=1,q\neq k}^{N_{\rm intr}} J_{\rm intr}^{(t)}(i,k,q)s^{(t)}(i,q)$$



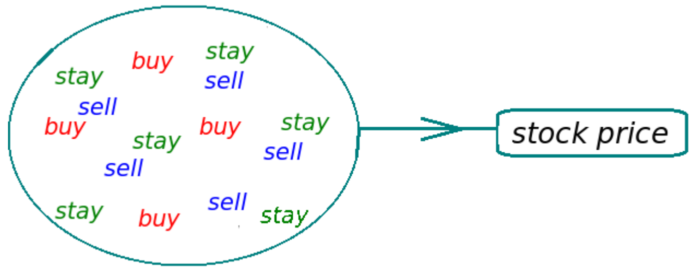
- **small-scale intra-agent** spin degrees of freedom, next decision

$$s^{(t)}(i,k) \leftarrow {\rm sign}\left(\; h_{\rm loc}^{(t)}(i,k) \;\right)\,, \qquad s^{(t)}(i,k) \in \{-1,1\}$$

- **large-scale** outer spin update → decision [sell...buy...stay]

$$S^{(t+1)}(i) \leftarrow \frac{1}{2}\left[\, s^{(t)}(i,1) + s^{(t)}(i,2)\,\right] \qquad S^{(t+1)}(i) \in \{-1,0,1\}$$

... magnetization related to log-price returns ...



$$\ln\left[p^{(t+1)}/p^{(t)}\right] = \frac{m^{(t)}}{\lambda}$$

$$m^{(t)} = \frac{1}{L}\sum_{i=1}^{L} S^{(t)}(i)$$

interpretation of $m^{(t)} > 0$ stock price increases 

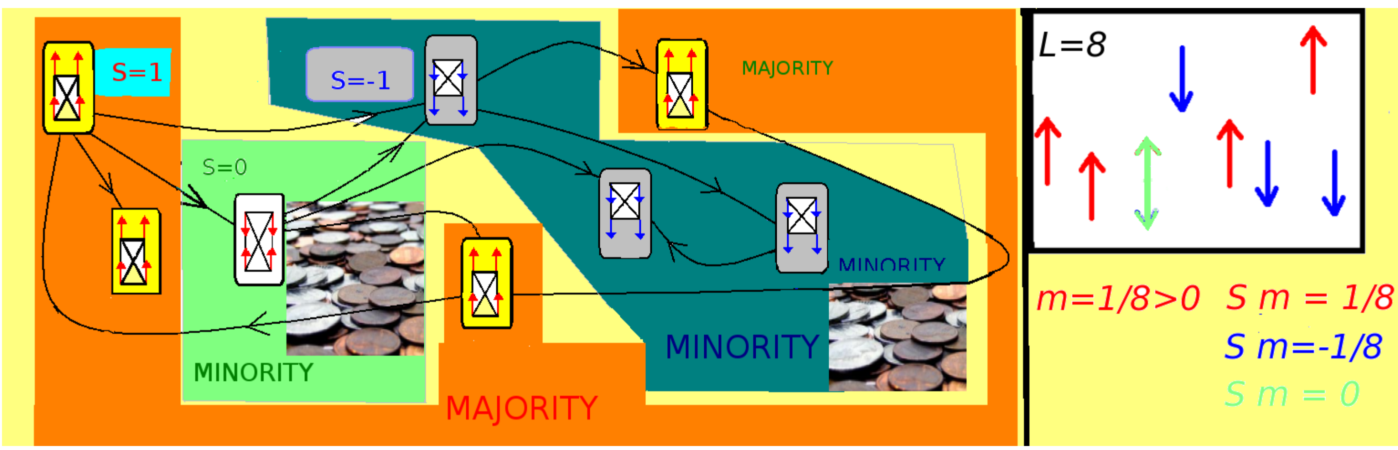
interpretation of $m^{(t)} < 0$ stock price decreases 

... local fitness and minority game concept ...

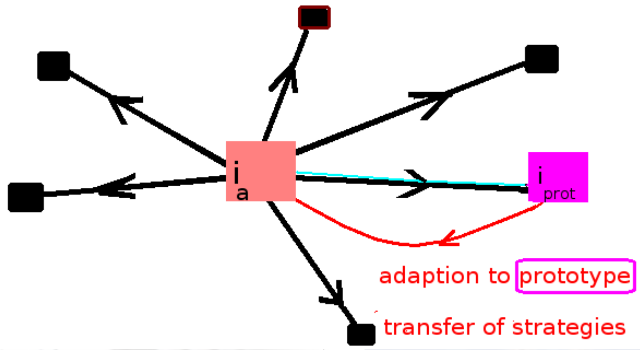
The valuation of agents according node fitness $F^{(t)}(i)$ that represents the integrated history of agent's gains and losses

$$F^{(t+1)}(i) = F^{(t)}(i) + S^{(t)}(i)\left[-c_0 m^{(t)} + c_{\mathrm{rand}} N^{(t)}(0,1)\right], \qquad c_0 > 0$$

- $S^{(t)}(i)$ is from the **minority**, its contribution is $S^{(t)}(i) m^{(t)} \leq 0$
- $S^{(t)}(i)$ is from the **majority**, its contribution is $S^{(t)}(i) m^{(t)} > 0$



..."slow" **adaptivity to prototype** (sub)leader...



- transfer of strategy from linked **prototype** (sub)leader

$$i_{\mathrm{prot}} \equiv X_n(i_{\mathrm{a}})\,, \qquad n \in I_{\mathrm{out}}$$

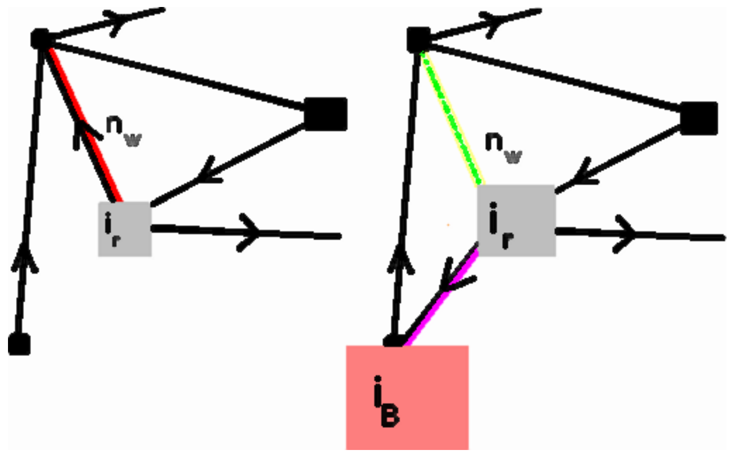
follower $i_{\mathrm{a}} \neq i_{\mathrm{prot}}$, $\forall i_{\mathrm{a}} \in \Gamma$:

$$J_{\mathrm{intr}}^{(t+1)}(i_{\mathrm{a}}, k, q) = J_{\mathrm{intr}}^{(t)}(i_{\mathrm{a}}, k, q)\,(1-\eta) + J_{\mathrm{intr}}^{(t)}(i_{\mathrm{prot}})\,\eta$$

plasticity parameter $\eta \in (0,1)$

...slow dynamics of network reconnections...

- **attachment preferences** [Barabási, Albert, Science **286** (1999) 509]
- i_{r} is connected **community leader** $i_{\mathrm{B}}(i_{\mathrm{r}})$
  [recommendation model- arXiv:nlin.AO/0609033]
- **reconnection**



- edge **disconnection** $X^{(t)}_{n_{\mathrm{W}}}(i_{\mathrm{r}})\,,\qquad n_{\mathrm{W}} \equiv \arg\min_{n \in I_{\mathrm{out}}} F(X^{(t)}_n(i_{\mathrm{r}}))$
- **connection** to the locally "**best**" node $i_{\mathrm{B}}\ (i_{\mathrm{r}}) \in \Gamma$

- recognize fitness on the set $\mathcal{F}(i_{\rm r}) \subset E_{N_{\rm rep}.N_{\rm path}}$:

$$\mathcal{F}(i_{\rm r}) \equiv \{\, F(i); \quad i \in \mathbf{RRW}(i_{\rm r})\,;\; i,\, i_{\rm r} \in \Gamma \,\}$$

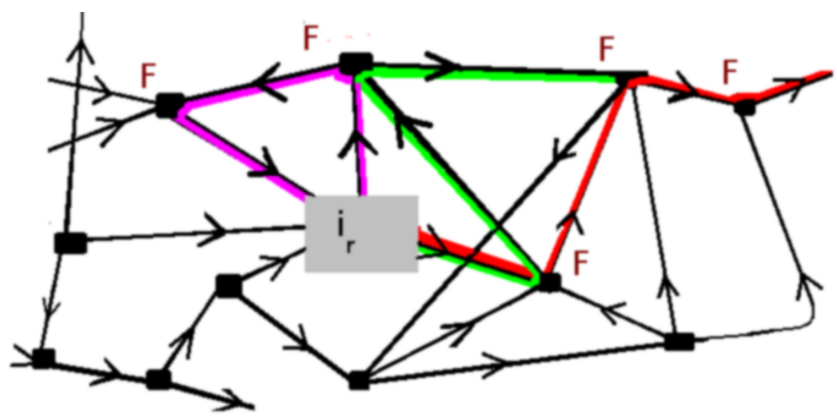
where

and

$$\mathbf{RRW}(i_{\rm r}) \equiv \cup_{z=1}^{N_{\rm rep}} \; \mathbf{path}_z(i_{\rm r})$$
$$\mathbf{path}_z(i_{\rm r}) \equiv \{\bar{i}_{1,z}(i_{\rm r}), \ldots, \bar{i}_{N_{\rm path},z}(i_{\rm r})\} \subset \Gamma\,, \qquad \bar{i}_p = X^{(t)}_{n_p}(\bar{i}_{p-1})\,, \qquad \bar{i}_0 \equiv i_{\rm r}$$

- find the best candidate for next connection according

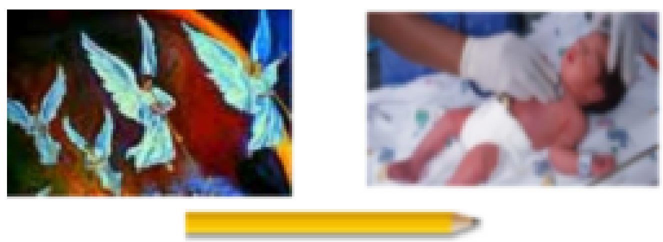
$$X_{n_{\rm W}}(i_{\rm r}) \leftarrow i_{\rm B} \equiv \arg\max \mathcal{F}(i_{\rm r})$$



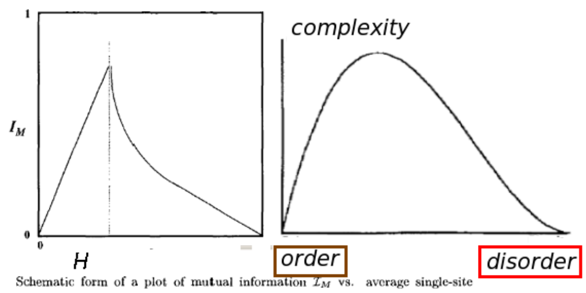
...extreme selective dynamics...

- localization of the weakest agent $i_{\min}$ that gains a minimum instant fitness $F^{(t)}(i_{\min})$
- death-birth rules: the strategies of weakest agent are replaced by Gaussian random N(.,.) quantities:

$$
\begin{aligned}
J_{\text{intr}}^{(t+1)}(i_{\min}) &\leftarrow \mathrm{N}^{(t)}(0, \sigma_{J_{\text{intr}}}) \\
F^{(t+1)}(i_{\min\mathrm{F}}) &\leftarrow \mathrm{N}^{(t)}(0, \sigma_{\mathrm{F}}) \\
s^{(t+1)}(i, k, q) & \quad \text{random from } \{-1, 1\}
\end{aligned}
$$



... metaoptimized parameters...



Schematic form of a plot of mutual information I_M vs. average single-site entropy H.

the requirements incorporated into multi-function optimization:
(a) closeness of the critical regime (SOC); [principle of extremely wide F distribution]
(b) highly leptocurtic log-price distribution; dynamics: bursts and boubles
(c) slow dynamics of strategies
(d) occasional reconnections

Bootstrapping (computing)

From Wikipedia, the free encyclopedia

For other uses of "bootstrapping", see bootstrapping.

In computing, **bootstrapping** refers to a process where a simple system activates another more complicated system that serves the same purpose. It is a solution to the Chicken-and-egg problem of starting a certain system without the system already functioning. The term is most often applied to the process of starting up a computer, in which a mechanism is needed to execute the software program that is responsible for executing software programs (the operating system).

The term "bootstrapping" alludes to a German legend about Baron Münchhausen, who claimed to have been able to lift himself out of a swamp by pulling himself up by his own hair. In later versions of the legend, he used his own boot straps to pull himself out of the sea which gave rise to the term bootstrapping[citation needed]. The term is believed to have entered computer jargon during the early 1950's by way of Heinlein's short story *By His Bootstraps* first published in 1941[citation needed].

- network topology: $L = 500,\ N_{\rm out} = 10$
- intranet topology: $N_{\rm intr} = 8$

- repeated random walk parametrized by $N_{\rm depth} = 6,\ N_{\rm rep} = 6$

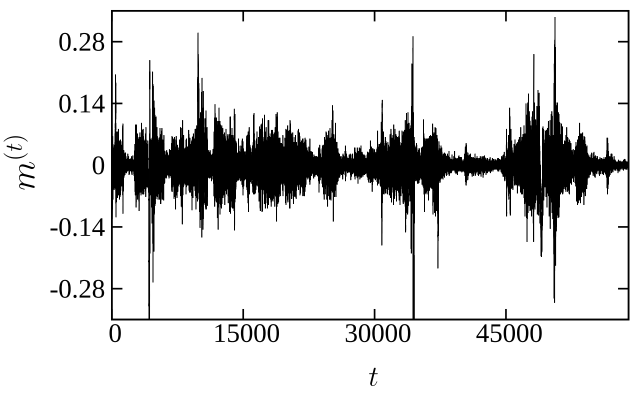
- dispersions of extremal dynamics:

$$\sigma_{J_{\rm intr}} = 1$$
$$\sigma_{\rm stoch} = 10^{-3},\ \sigma_{\rm F} = 0.1;$$

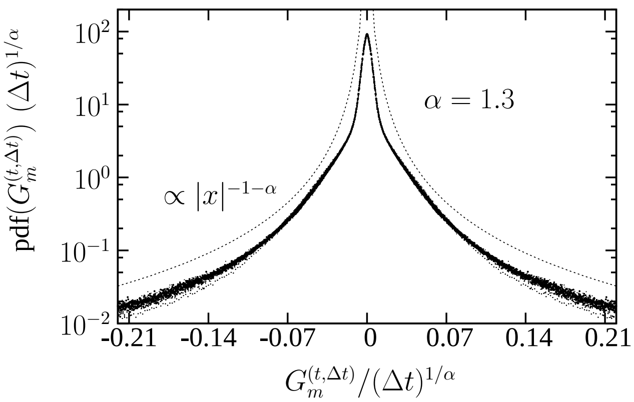
- probabilities of applying of adaptivity and reconnection $P_{\rm Re} = 0.01,\ P_{\rm Ad} = 0.2;$
- fitness $c_0 = 1,\ c_{\rm ran} = 0$

- adaptivity: $\eta = 0.1$

... statistics, scaling of price returns ...



(a)



(b)

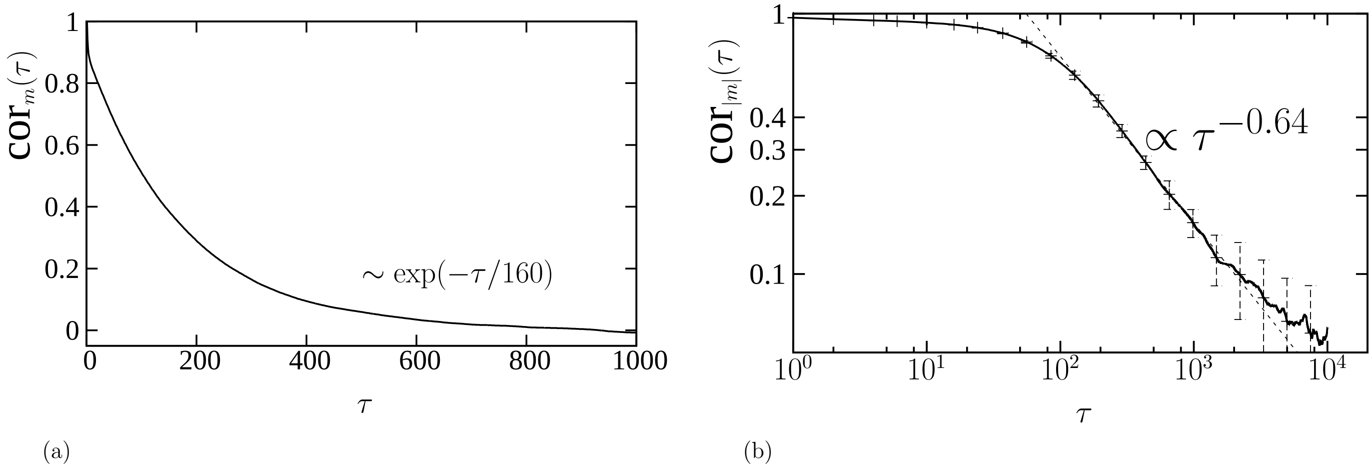
(a) time evolution of the log-price returns defined as magnetization $m^{(t)}$.
(b) shows the scaling properties of the pdf of cumulated returns

$$G_m^{(t,\Delta t)} = \sum_{k=0}^{\Delta t} m^{(t+k)}$$

for $\alpha = 1.3$ scaling collapse of eight distributions: $\Delta t = 1, 2, \ldots, 8$..

$$\text{pdf}(G_m^{(t,\Delta t)}) = (\Delta t)^{-1/\alpha}\, \Phi\left(\frac{G_m^{(t,\Delta t)}}{(\Delta t)^{1/\alpha}}\right), \qquad \Phi(x) \propto |x|^{-1-\alpha}$$

... identification of basic time scale ...



(a) the fastest changes belong to $m^{(t)}$ memory

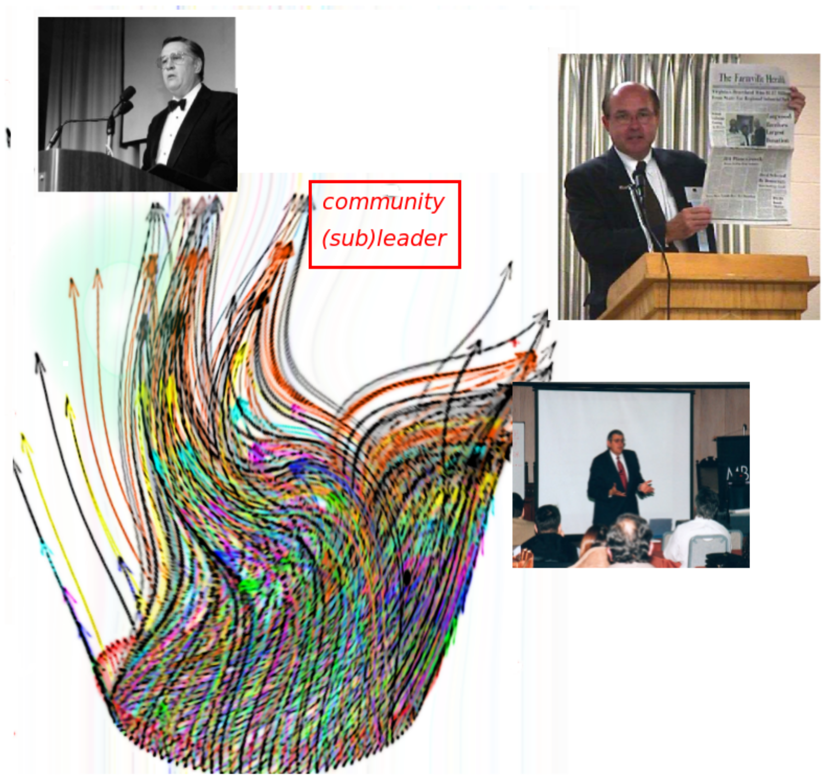
(b) volatility $|m|$ defines the long-time memory identified by autocorrelation function

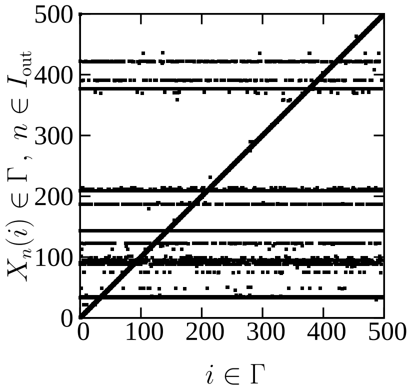
$$\mathrm{cor}_{|m|}(\tau) = \frac{\langle |m(\tau)||m(0)|\rangle - \langle |m(\tau)|\rangle\langle |m(0)|\rangle}{\langle |m(0)|^2\rangle - \langle |m(0)|\rangle^2} \sim \tau^{-0.64}$$

(c) "slowest" time scale belong to fitness (wealth accumulation)

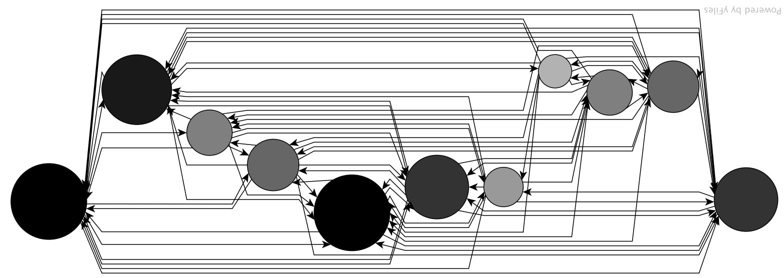
... network of distributed (sub)leaders...

- snapshot of the **community structure** on the complex network;

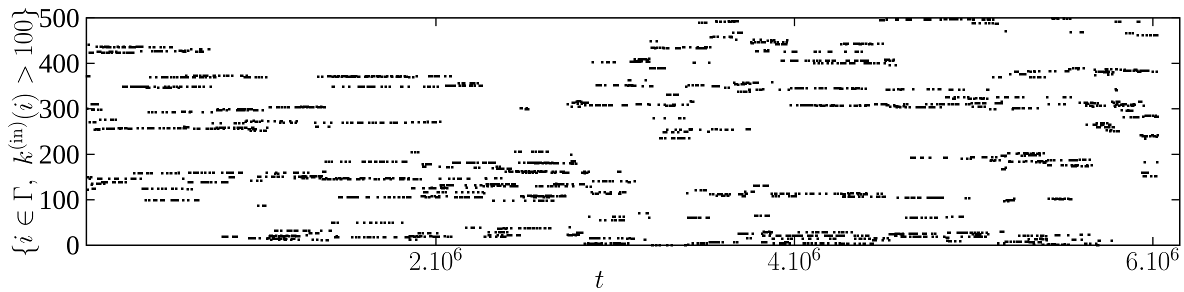




(a)



(b)



(c)

(a) The snapshot of adjacency $L \times L$ matrix. The cyclic L-gon maps onto the matrix diagonal.

(b) The subgraph $\{i \in \Gamma, k^{(\rm in))} > 50\}$ (the larger circle belongs to larger $k^{(\rm in)}$.

(c) The epochs of topology monitored by means of selection of highly preferred

$$\{i,\, i \in \Gamma,\, k^{(\rm in)}(i) > 100\,\} \subset \Gamma$$

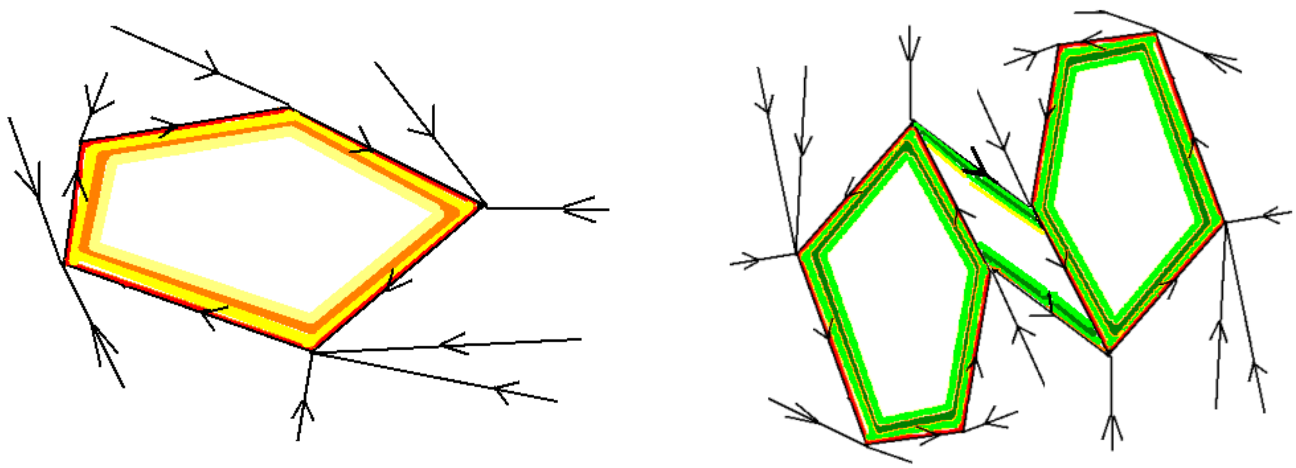
.

The members of such group may be vaguely identified as local leaders.

- network recognized by random walker exhibits attractors (preferential paths-closed loops) linking (sub)leaders
- generalized **topological** Ljapunov-like exponents
- how distance of the pair of diffusing agents evolve?:

$j_{p+1} = X_{n_p}(j_p)$, $i_{p+1} = X_{m_p}(i_p)$, $n_p, m_p \in I_{\rm out}$, $p = 1, \ldots, M$

$$\lambda_{(M)} = \frac{1}{M}\left\langle \ln\left[\frac{l(i_M, j_M)}{l(i_0, j_0)}\right]\right\rangle \qquad (\lambda_{(M)} < 0)$$



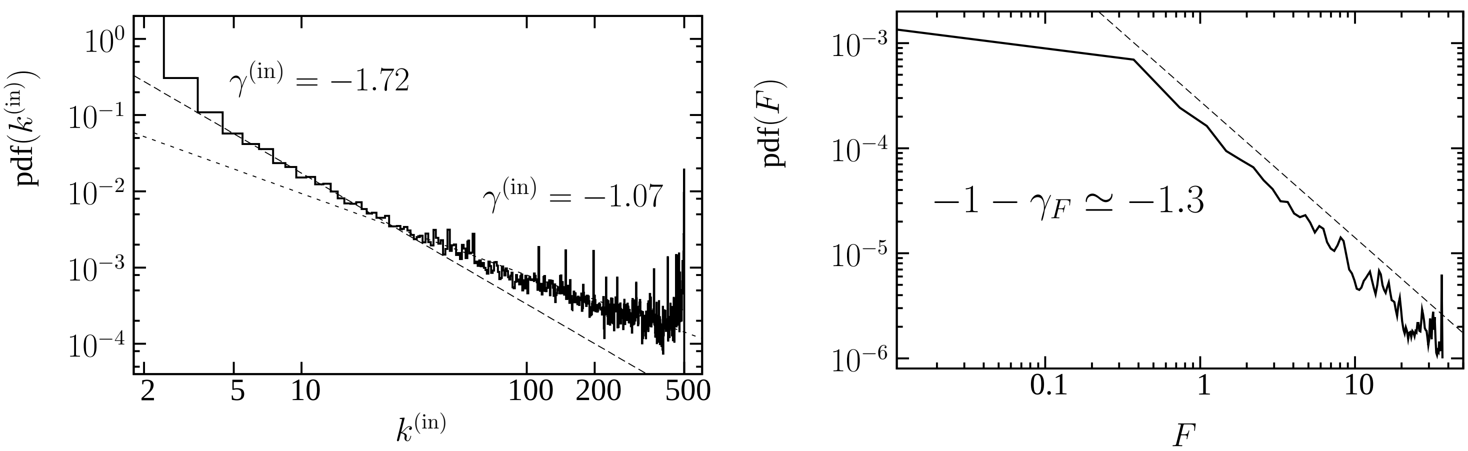
... statistics of fitness and node degrees ...

- power-law distribution of fitness related to **Pareto-Zipf** 0.3 law
- topological consequence: power-law distribution of incoming node degrees,

  (here $\gamma^{\text{in}} = -1.72$) [Barabási, Albert, Science **286** (1999) 509.]

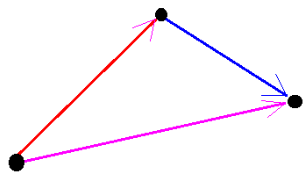
[Y.Fujiwara et.al, Physica A, 334 (2004) 112.]

French firms in 2001 {Pareto index $=-1.84$};
UK firms (2000/2001) {Pareto index -1.995}



(a) (b)

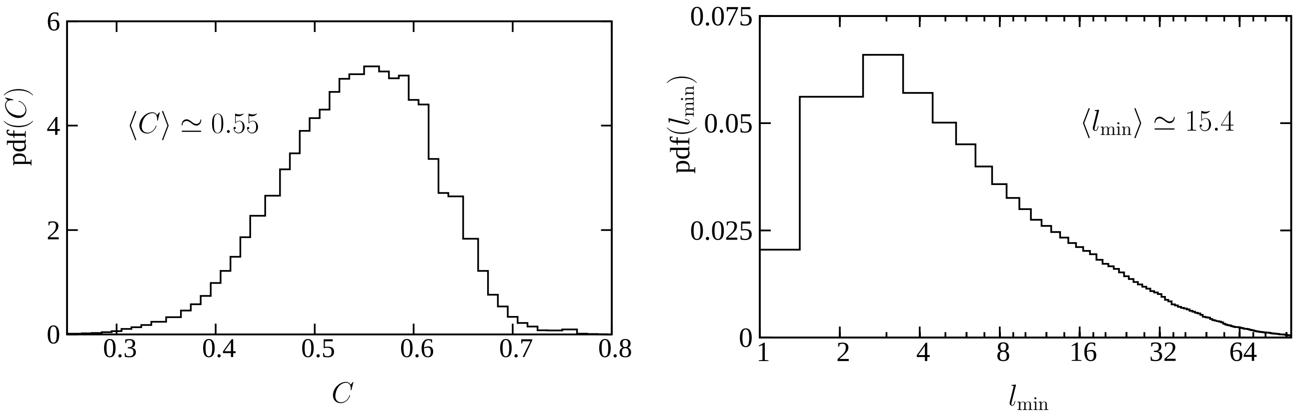
... clustering coefficient ...



statistical measure of social transitivity

$$C(i) = \frac{e(i)}{N_{\rm out}(N_{\rm out}-1)}\,, \qquad e(i) \equiv \sum_{n_1,n_2,n_3=1}^{N_{\rm out}\times N_{\rm out}\times N_{\rm out}} \delta_{X_{n_1}(X_{n_2}(i)),X_{n_3}(i)}$$

for partially random net $\langle C_{\rm random}\rangle \simeq 0.02$
our complex net $\langle C\rangle \sim 0.51$



(a) (b)

Conclusions

- the minimalist spin market model with **strategic variables self-adjusted via the co-evolutionary competitive changes and evolving topology has been suggested.**

- it has been demonstrated that the **extremal selective dynamics** induces the **power-law** distributions in analogy with **Bak-Sneppen** model [P. Bak and K. Sneppen, "Punctuated equilibrium and criticality in a simple model of evolution" Phys. Rev. Lett. 71, 4083X4086 (1993)].

- the pdf's of log-price returns fall to the class of **fat-tailed** distributions that are observed in data of economic origin.

- the **long-time memory of the autocorrelation** function of **volatility** of the log-price returns has been detected.

- the "**small world**" behavior [including high $\langle C \rangle$, power-law of pdf($k^{(\mathrm{in})}$) combined with sufficiently small $\langle l_{\min} \rangle$] has been observed that **is symptomatic for social networking**.